Finding the reflection coefficient of a differential one-port device

Is measuring the reflection coefficient of ICs causing you problems? This new method can help.

By Lutz Konstroffer

For noise immunity reasons, semiconductor devices used in mobile phones often have differential input and output ports. However, when it comes to the measurement of the reflection coefficient, differential ports are a frequent source of headaches for the designer. This is caused by the high integration of modern integrated circuit (IC) devices. In many of these devices, access is given to only one input (or output) port but not both. However, external components such as surface acoustic wave (SAW) filters, require well defined reflection coefficients at their inputs and outputs. Because of this, wireless engineers often need to measure the complex reflection coefficient of a differential one-port.

The typical way of measuring the complex reflection coefficient of a differential one-port is by using a network analyzer with a balun or transformer. However, this method introduces uncertainty caused by the requirement of self-made calibration standards, parameter variations of the transformer and temperature problems. A new method of determining the reflection coefficient of a differential one-port is to measure the S-parameters of an equivalent single-ended two-port.

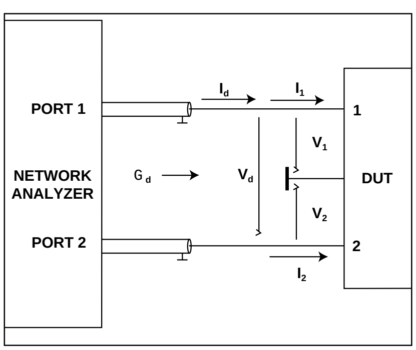

Figure 1. Splitting the balanced one-port into an equivalent two-port with single-ended ports.

The method

The two wires of a differential one-port can be regarded as single-ended input and output ports of an equivalent two-port. The S-parameters of this equivalent single-ended two-port are easy to measure with a network analyzer. The problem remains of how to convert the S-parameters of the equivalent two-port into the complex reflection coefficient of the differential one-port. Figure 1 shows how the voltages and currents of the equivalent two-port relate to those of the differential one-port.

From Figure 1, it can be seen that:

$$V_d = V_1 - V_2 \quad (1)$$

$$I_d = I_1 = -I_2 \quad (2)$$

With the incoming waves a_1 and a_2 and the outgoing waves b_1 and b_2, the voltage V_1 is:

$$V_1 = \sqrt{Z_o}\left(a_1 + b_1\right) \quad (3)$$

Z_0 is the characteristic impedance of the system (the impedance of the network analyzer), to which the measured S-parameters of the device under test (DUT) refers.

With $b_1 = S_{11}a_1 + S_{12}a_2$ it follows that:

$$V_1 = \sqrt{Z_o}\left(a_1 + S_{11}a_1 + S_{12}a_2\right) \quad (4)$$

Similarly the current I_1 is:

$$I_1 = \frac{1}{\sqrt{Z_o}}\left(a_1 - b_1\right) \quad (5)$$

Substituting for b_1 gives:

$$I_1 = \frac{1}{\sqrt{Z_o}}\left(a_1 - S_{11}a_1 - S_{12}a_2\right) \quad (6)$$

The same equations apply to V_2 and I_2:

$$V_2 = \sqrt{Z_o}\left(a_2 + S_{22}a_2 + S_{21}a_1\right) \quad (7)$$

$$I_2 = \frac{1}{\sqrt{Z_o}}\left(a_2 - S_{22}a_2 - S_{21}a_1\right) \quad (8)$$

Substituting into Equation 2 gives:

$$a_1 - S_{11}a_1 - S_{12}a_2 = -\left(a_2 - S_{22}a_2 - S_{21}a_1\right) \quad (9)$$

Solving Equation 9 for a_2 gives:

$$a_2 = -a_1\frac{1 - S_{11} - S_{21}}{1 - S_{22} - S_{12}} \quad (10)$$

Equation 10 determines all the voltages and currents in terms of the incoming wave a_1 only.

From Equations 1, 4 and 7, it follows that:

$$\frac{V_d}{\sqrt{Z_o}} = a_1\left(1 + S_{11} - S_{21}\right) - a_2\left(1 + S_{22} - S_{12}\right) \quad (11)$$

Using Equation 10 gives:

$$\frac{V_d}{a_1\sqrt{Z_o}} = \left(1 + S_{11} - S_{21}\right) + \frac{\left(1 + S_{22} - S_{12}\right)\left(1 - S_{11} - S_{21}\right)}{\left(1 - S_{22} - S_{12}\right)} \quad (12)$$

Similarly, substitution of Equation 10 into Equations 6 and 2 gives:

$$\frac{I_d\sqrt{Z_o}}{a_1} = 1 - S_{11} + S_{12}\frac{\left(1 - S_{11} - S_{21}\right)}{\left(1 - S_{22} - S_{12}\right)} \quad (13)$$

The incoming and outgoing waves a_d and b_d of the differential port are related to V_d and I_d by the normalized Heaviside transformation:

$$a_d = \frac{1}{2}\left(\frac{V_d}{\sqrt{Z_o}} + I_d\sqrt{Z_o}\right) \quad (14)$$

$$b_d = \frac{1}{2}\left(\frac{V_d}{\sqrt{Z_o}} - I_d\sqrt{Z_o}\right) \quad (15)$$

Inserting Equations 12 and 13 into Equation 14 gives:

$$\frac{a_d}{a_1} = \frac{1}{2}\left[2 - S_{21} + \frac{\left(1 - S_{11} - S_{21}\right)\left(1 + S_{22}\right)}{\left(1 - S_{22} - S_{12}\right)}\right] \quad (16)$$

The expression for b_d will be:

$$\frac{b_d}{a_1} = \frac{1}{2}\left[2S_{11} - S_{21} + \frac{\left(1 - S_{11} - S_{21}\right)\left(1 + S_{22} - 2S_{12}\right)}{\left(1 - S_{22} - S_{12}\right)}\right] \quad (17)$$

Dividing Equation 17 by Equation 16

directly gives the reflection coefficient G_d of the differential one-port:

$$\Gamma_d = \frac{2S_{11} - S_{21} + \dfrac{(1 - S_{11} - S_{21})(1 + S_{22} - 2S_{12})}{(1 - S_{22} - S_{12})}}{2 - S_{21} + \dfrac{(1 - S_{11} - S_{21})(1 + S_{22})}{(1 - S_{22} - S_{12})}} \quad (18)$$

or:

$$\Gamma_d = \frac{(2S_{11} - S_{21})(1 - S_{22} - S_{12}) + (1 - S_{11} - S_{21})(1 + S_{22} - 2S_{12})}{(2 - S_{21})(1 - S_{22} - S_{12}) + (1 - S_{11} - S_{21})(1 + S_{22})} \quad (19)$$

Equation 19 transforms the easy to measure S-parameters of the equivalent two-port formed by the single-ended ports 1 and 2 into the complex reflection coefficient G_d of the differential one-port.

This result was successfully proven by both practical measurements and computer simulations.

Conclusion

The reflection coefficient of a differential one-port, such as a input or output of a semiconductor device, can be measured without any baluns or transformers. The following steps must be completed:

1. Regard the two wires (for instance, the pins of the semiconductor device) as single-ended ports of an equivalent two-port. Measure the S-parameters of this two-port using a network analyzer. The network analyzer can be calibrated either with self-made single ended standards or with the standards provided by the manufacturer and the appropriate choice of a delay time (port extension). For open collector outputs, use the network analyzer's internal bias-tee.

2. Calculate the reflection coefficient of the differential one-port from the results of the S-parameter measurement performed in step 1 using Equation 19. **RF**

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